$^{40}Ar/^{39}Ar$ Thermochronology: Diffusion and Dodson

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Helsinki,

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Learning Outcomes

- You will gain an understanding of:
  - Diffusion in minerals and rocks
  - Closure temperature
  - The assumptions behind the Dodson T_C formulation

- You will be able to:
  - Calculate closure temperatures

Diffusion

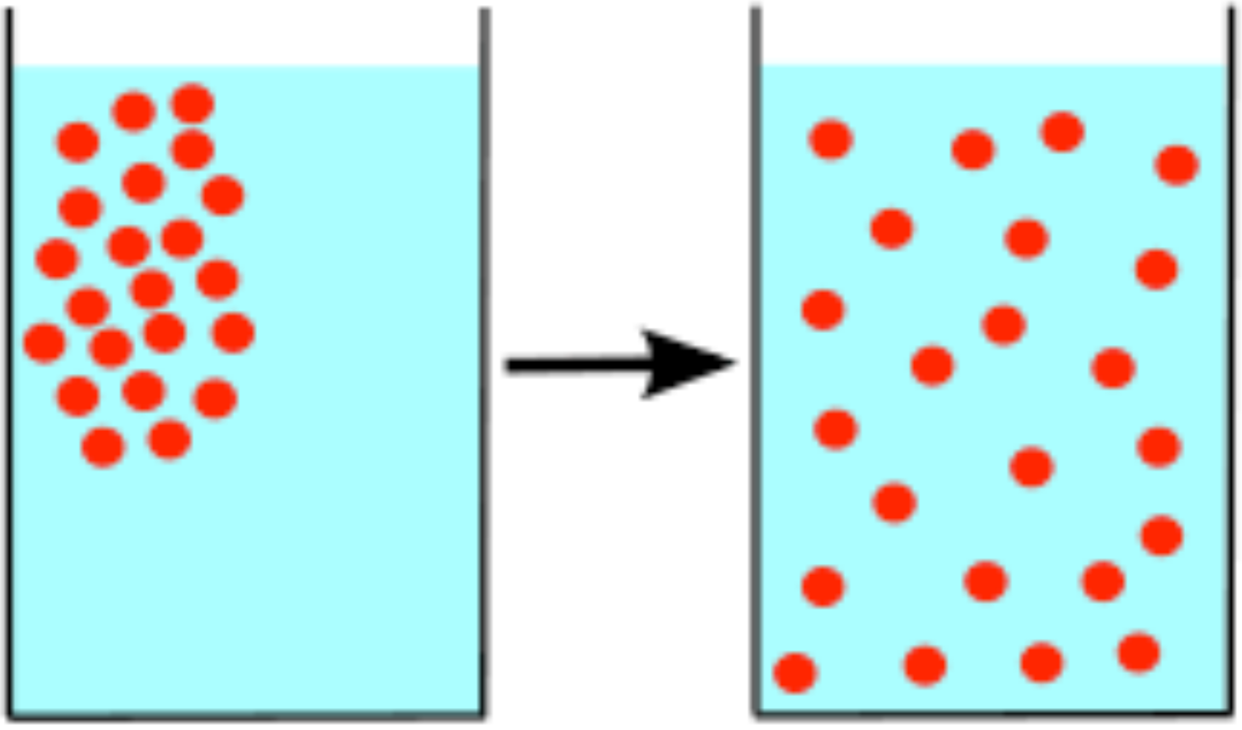


Image: Wikipedia

Diffusion in Minerals

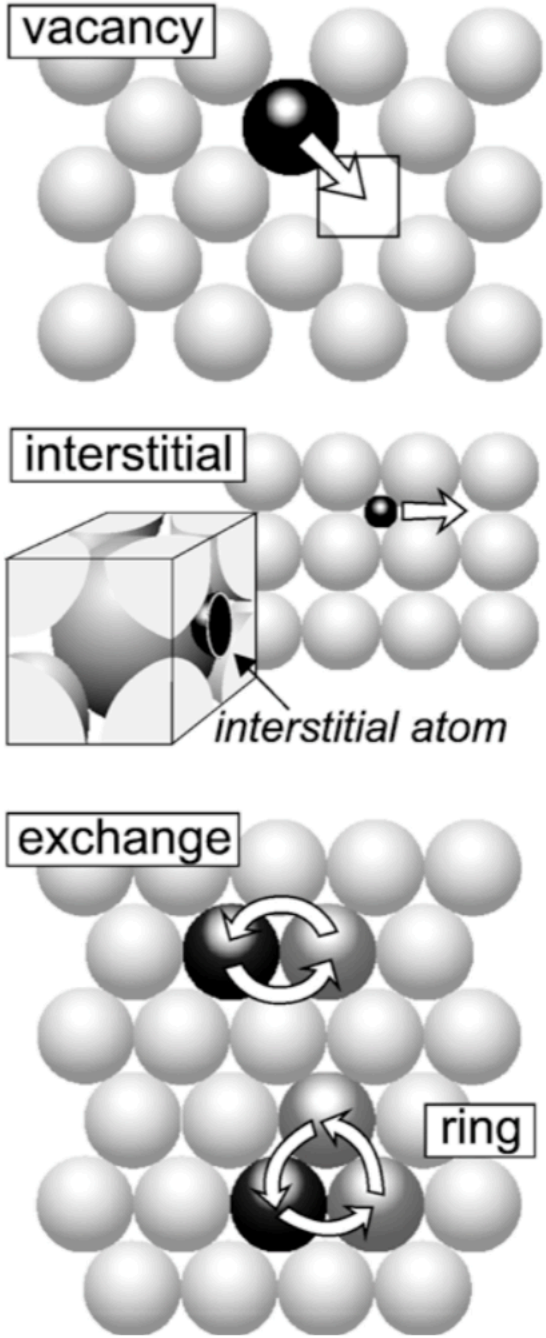
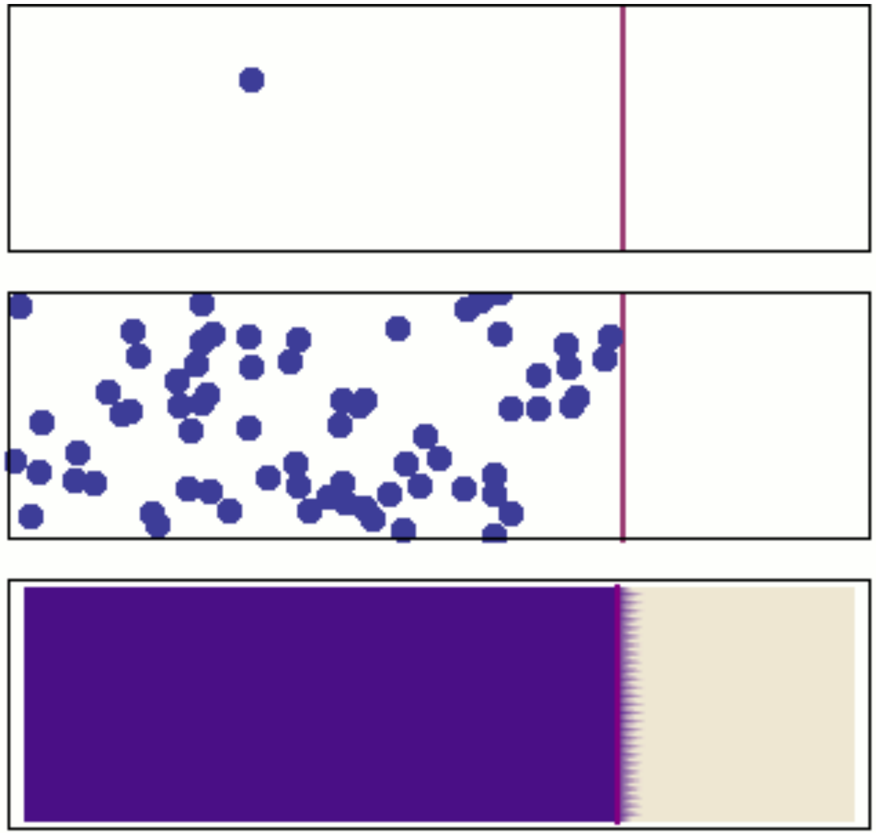


Image: Watson + Baxter 2010

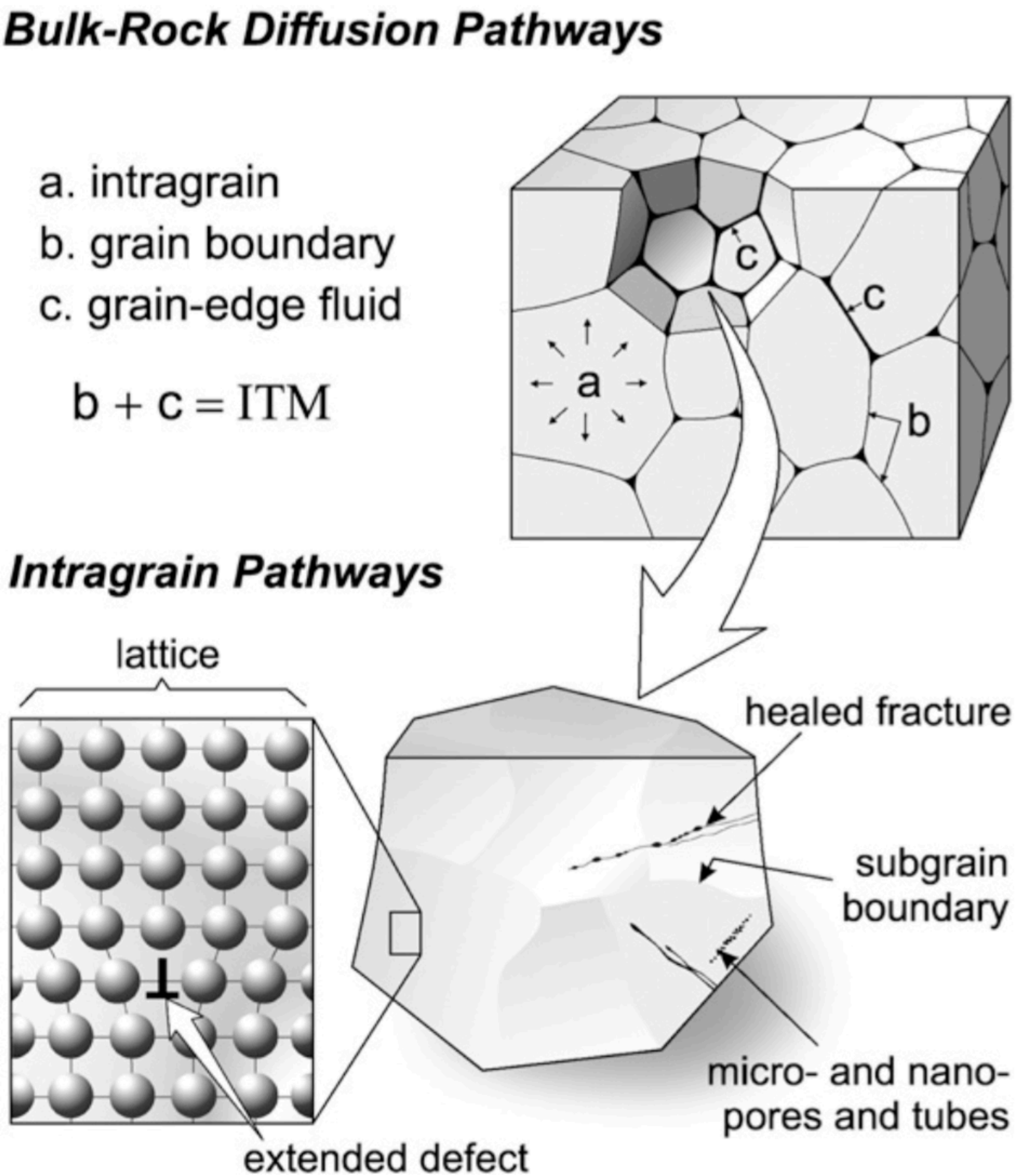


Low concentrations: random walk

High concentrations: smooth and deterministic

By Sbyrnes321:, https://commons.wikimedia.org/w/index.php?curid=8995324

Diffusion in Rocks

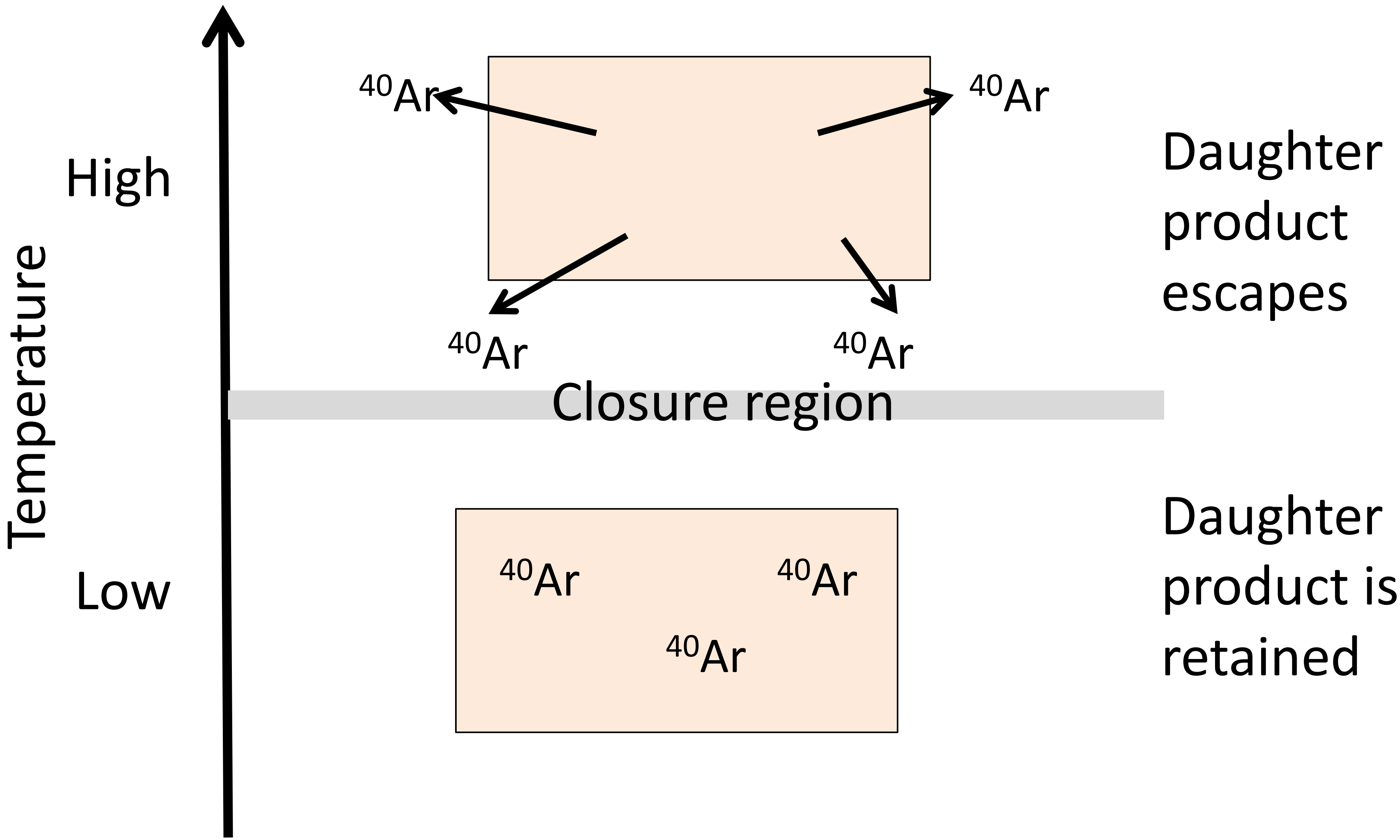


Not volume diffusion

Could be barriers or pathways

Image: Watson + Baxter 2010

Closure Temperature



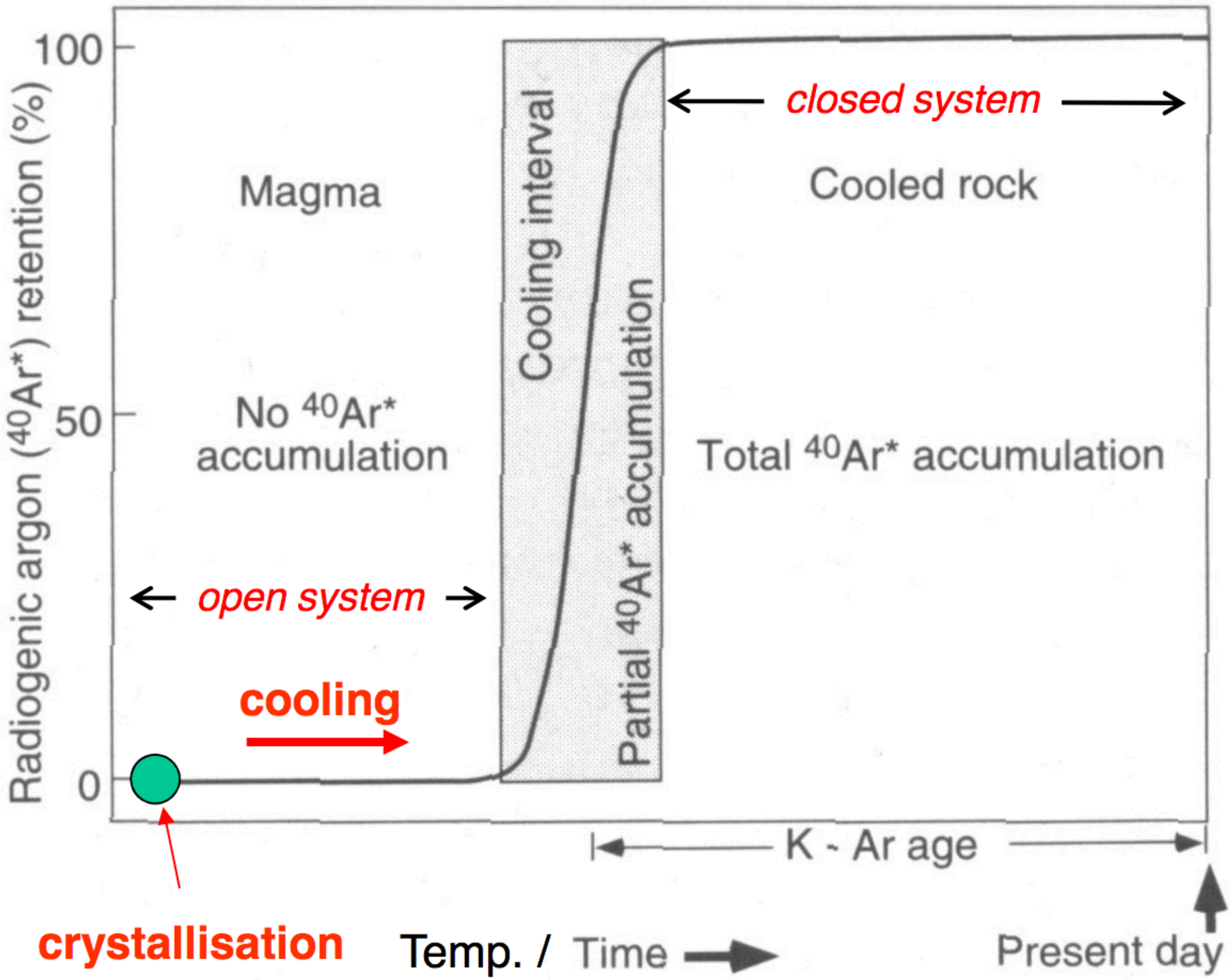


Image: http://studylib.net/doc/18050406/ar-ar-geo--thermochronology

Fick's 2nd law of diffusion

$$\frac{\partial c}{\partial t} = D\nabla^2 c + S$$

This equation describes the concentration (of argon) in 3 dimensions as a function of space and time

C= concentration

D = diffusion coefficient

∇^2 = (x,y,z directions)

S = source term (Ar is being created over time)

Linking age to T: Dodson T_c

$$\frac{\partial c}{\partial t} = D\nabla^2 c + S$$

Note: Mistake in abstract of Dodson paper: E_a and R transposed!

$$T_c = E_a / [R \ln(A\tau D_0 / a^2)]$$

$$\tau = RT^2/(E\, dT/dt)$$

Note: dT/dt must be decreasing

R = gas constact

E_a = activation energy

A = shape factor (line, plane, sphere)

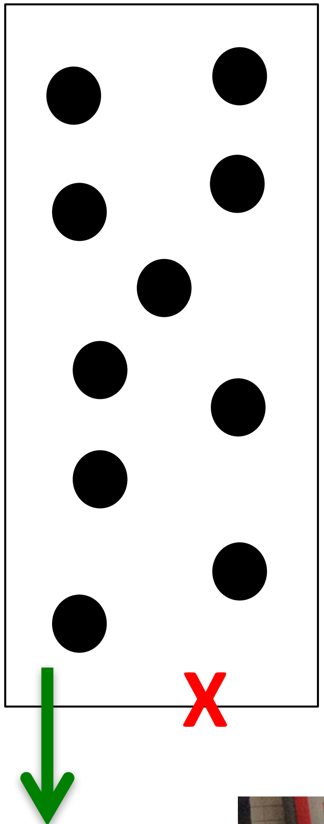
τ = time constant with which D diminishes with T

D_0 = diffusion coefficient

a = diffusion radius

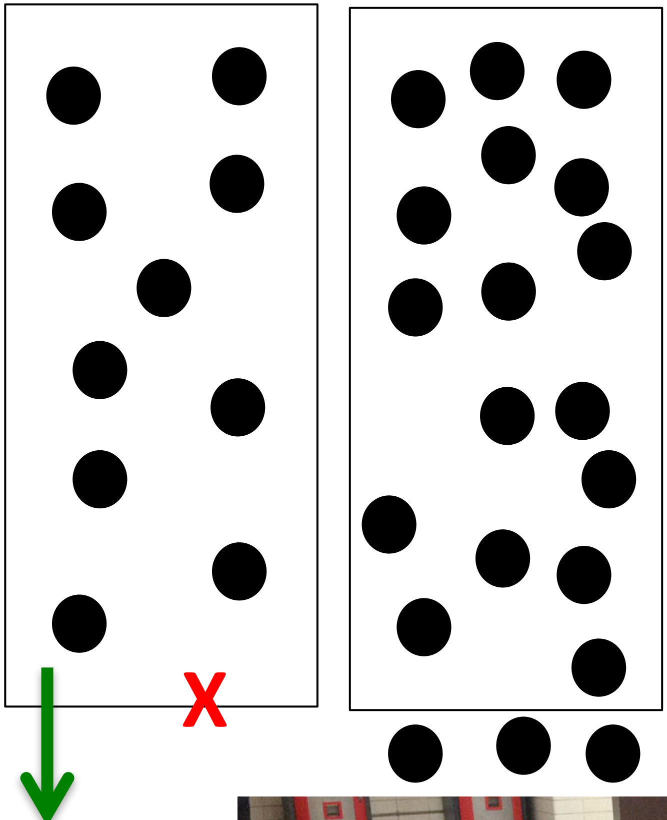
Dodson, CMP, 1973

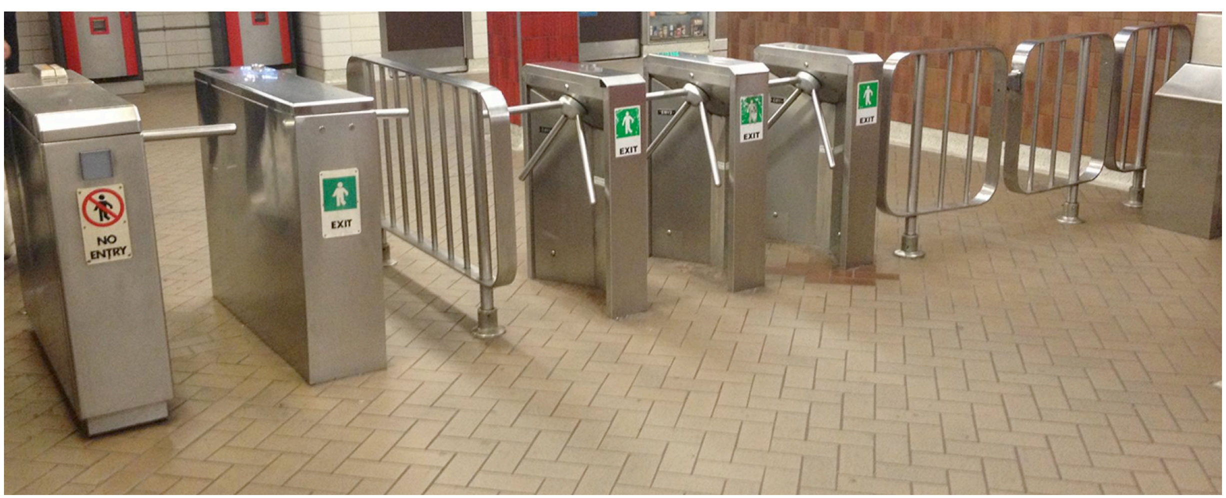
10

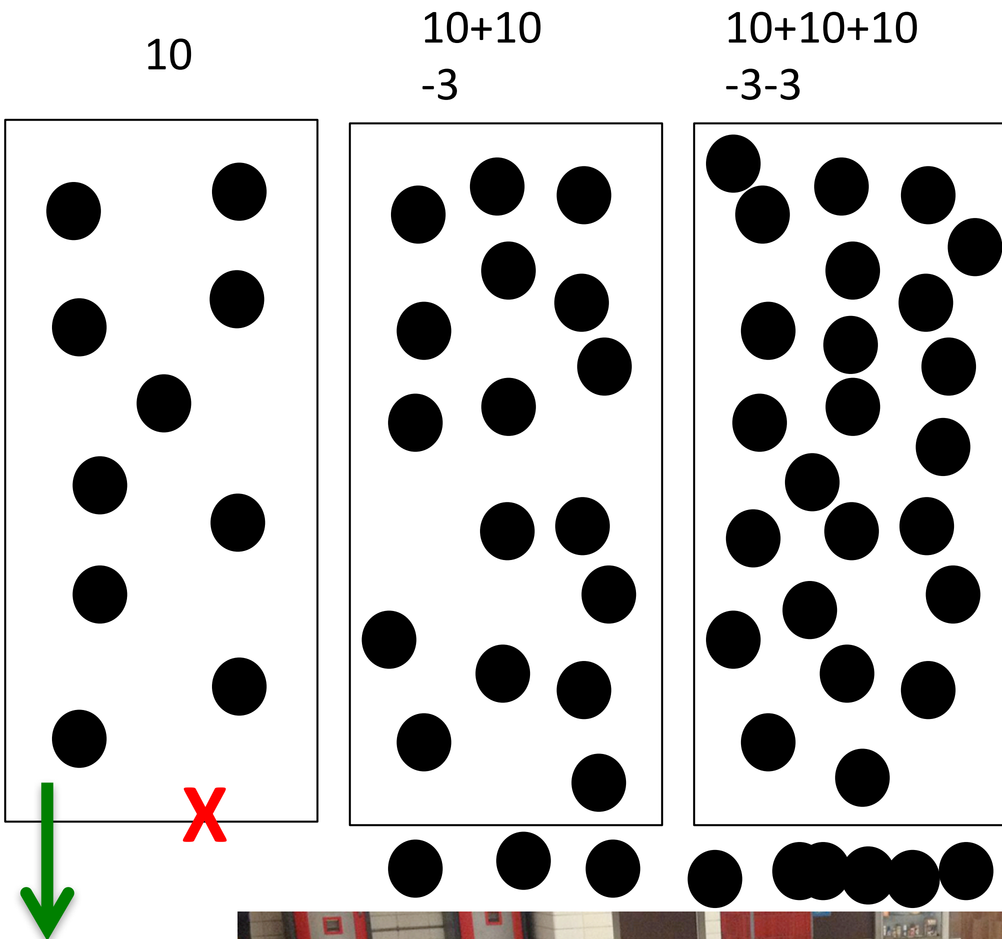


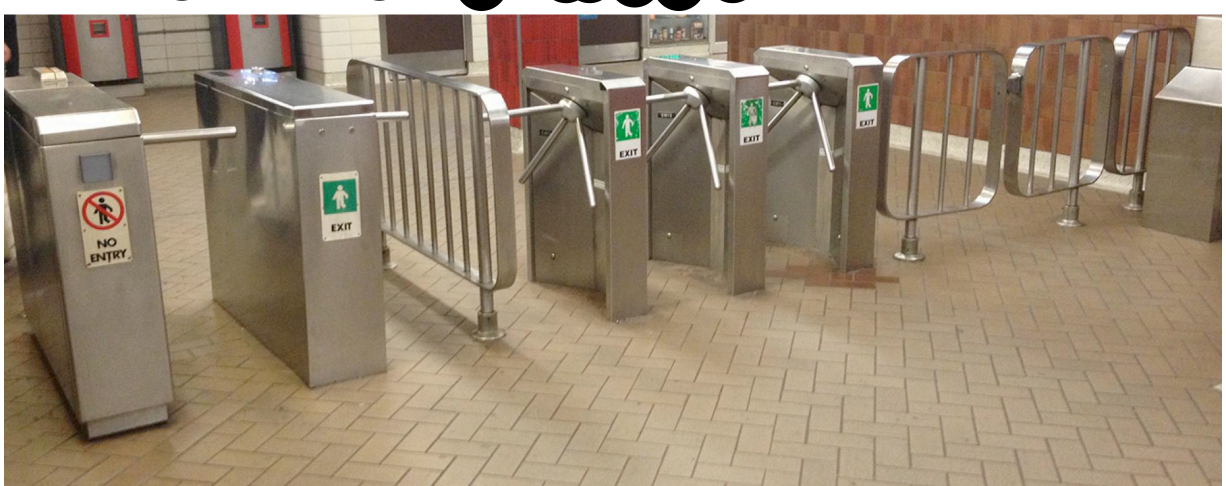
10

10+10
-3






10
10+10
-3
10+10+10
-3-3
X


NO
ENTRY
EXIT

10

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-3

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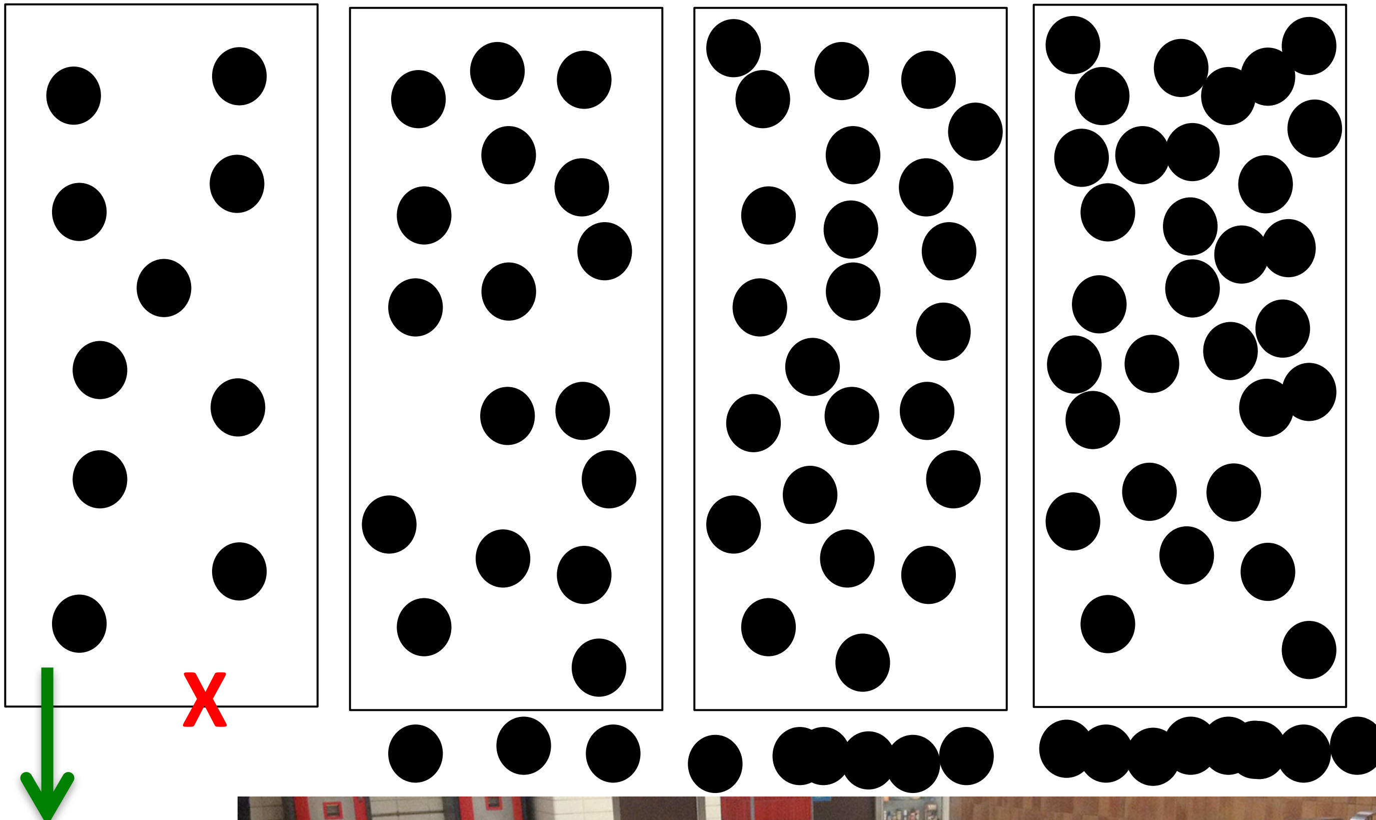
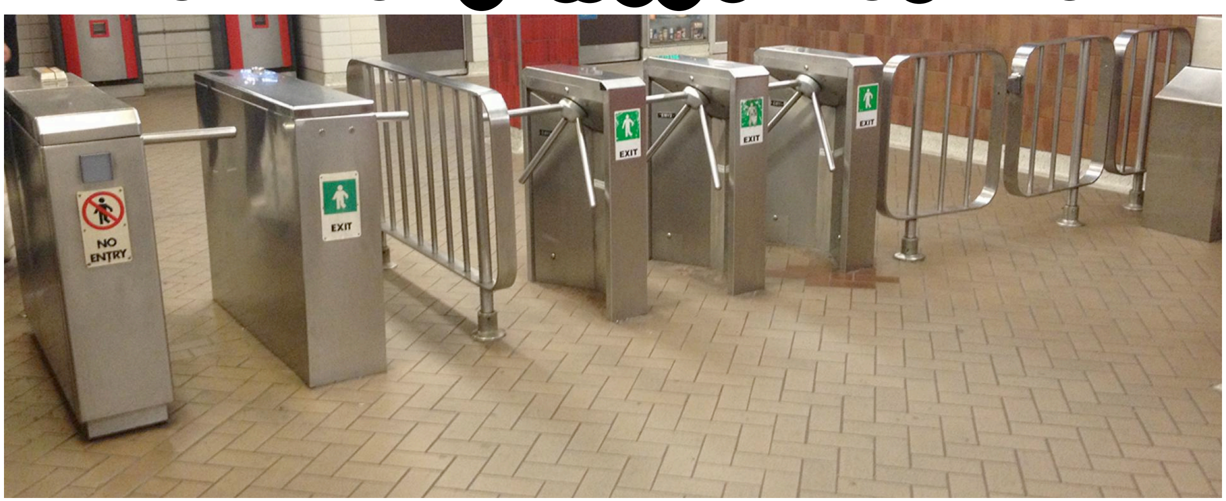
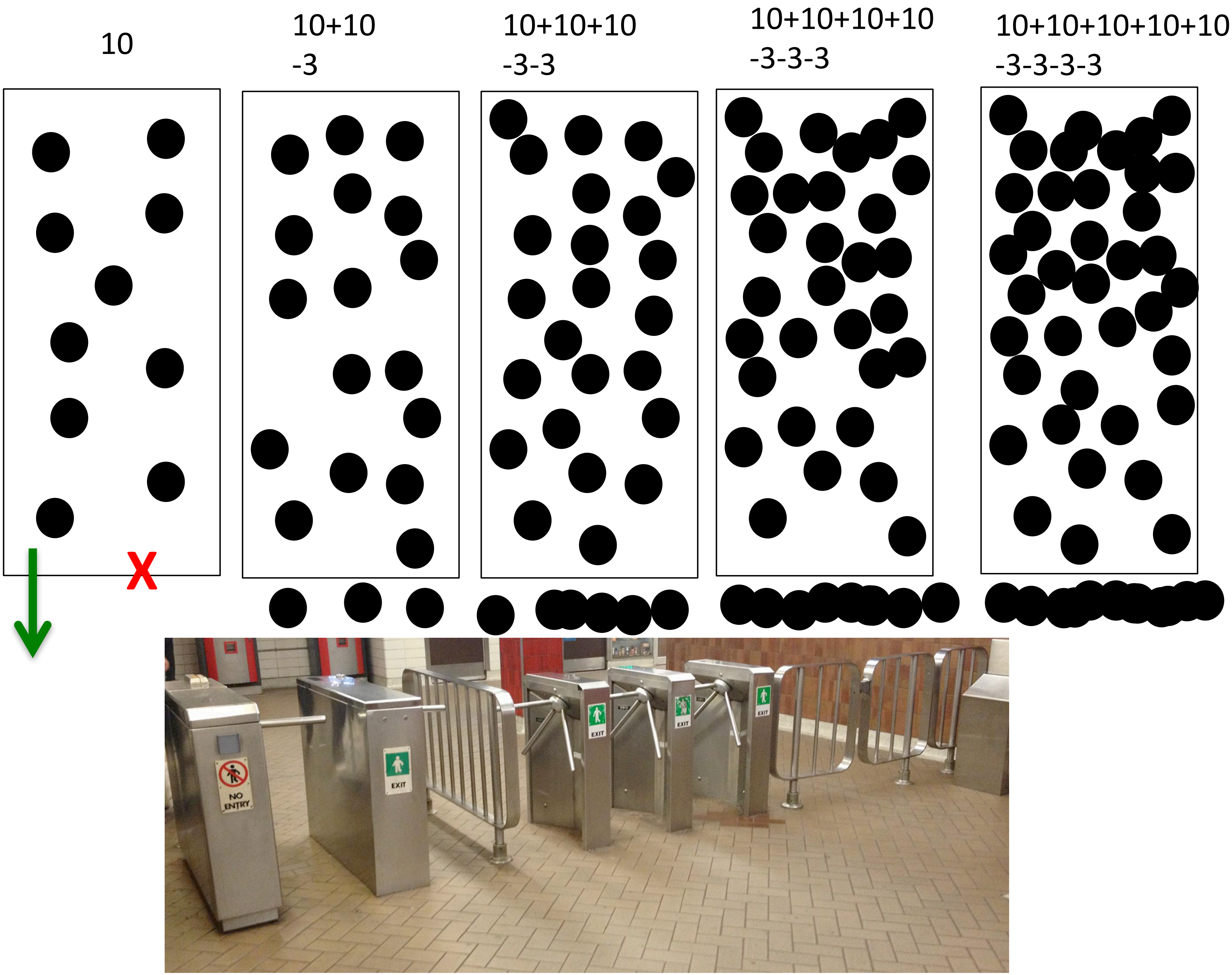
10+10+10+10
-3-3-3



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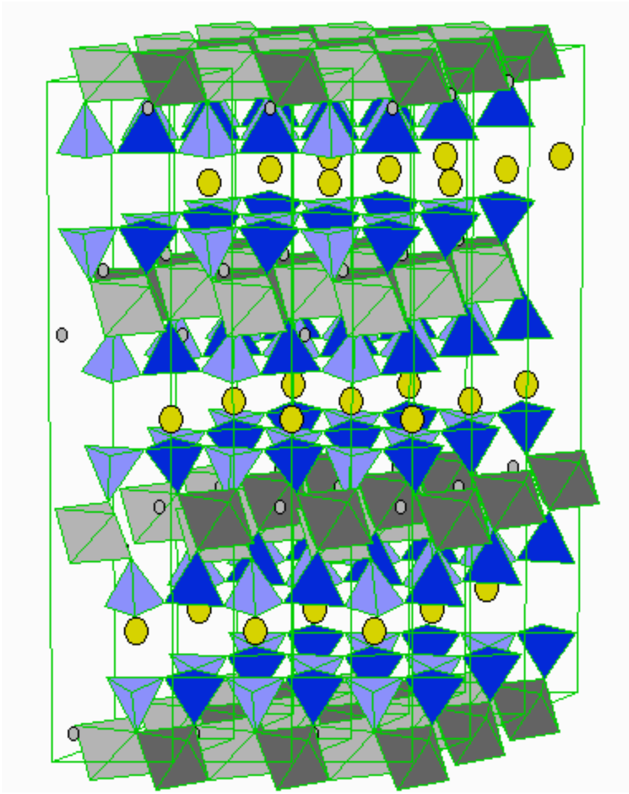
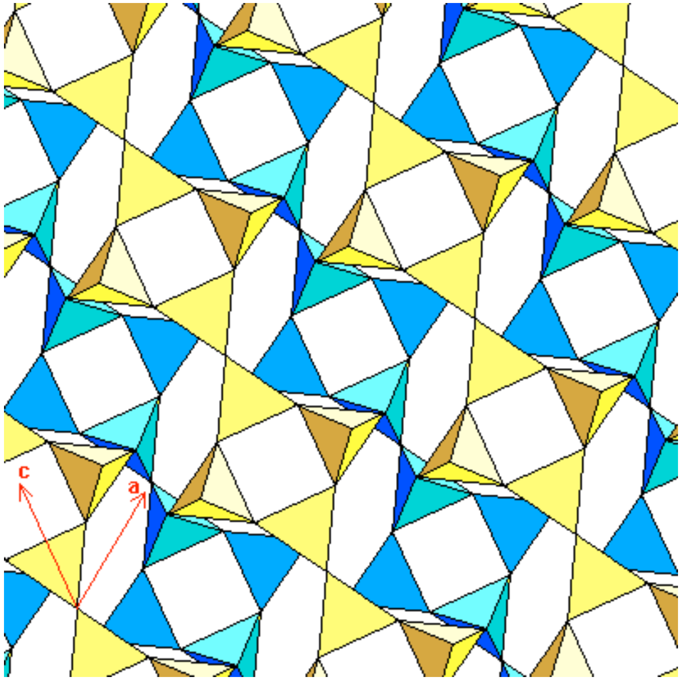
X
NO ENTRY
EXIT
EXIT
EXIT
EXIT

Remember minerals have different lattice properties



Mica: cylinder (2D)



Feldspar: sphere (3D)

Images: ruby.colorado.edu and uwgb.edu

Create a T_c Spreadsheet for muscovite

Constants:

$R = 1.986 \times 10^{-3}$ kcal/K/mol

$D_0 = 2.3\ cm^2\ s^{-1}$ (Harrison et al., 2009)

$E_A = 63\ kcal\ mol^{-1}$

A = 27 for cylinder

$$T_c = E_a / [R \ln(A \tau D_0 / a^2)]$$

$$\tau = RT^2/(E_a\ dT/dt)$$

Let's use a grain radius of 0.05 cm

And a cooling rate of $10\ K\ Ma^{-1}$ (convert to $K\ s^{-1}$)

What is T_C?

Hint: you will need to iterate. Choose a starting temperature to calculate τ, then iterate from that value

Now calculate and plot a graph of:

activity
TIME

- Closure T for muscovite for cooling rates from 0.01 to 100 K Ma^{-1}
- For grain radii of 0.001 to 1 cm

Well done!

- You can use this spreadsheet to calculate Tc of:
- Ar in other minerals
- He in other minerals
- Pb in other minerals….
- Basically diffusion of any radiogenically-produced element in any mineral.

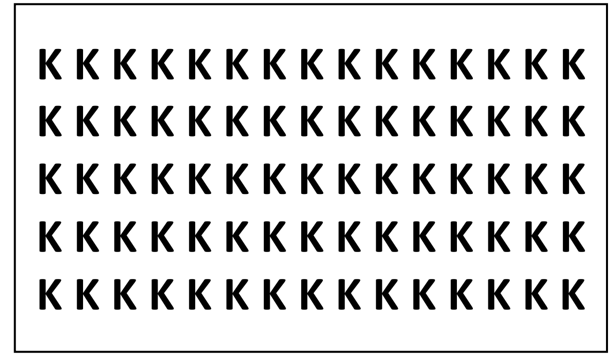
Mineral	D_0 (cm^2 s^{-1})	E_a (kJ mol^{-1})	Reference
Phlogopite (Ann_4)	$0.75^{+1.7}_{-0.52}$	242 ± 11	Giletti (1974)
Biotite (Ann_{56})	$0.077^{+0.21}_{-0.06}$	196 ± 9	Harrison et al. (1985)
Biotite (Ann_{56})	$0.015^{+0.022}_{-0.005}$	188 ± 12	Grove and Harrison (1996)
Biotite (Ann_{56})	$0.075^{+0.049}_{-0.021}$	197 ± 6	Combined data of Harrison et al. (1985) and Grove and Harrison (1996)
Biotite (Ann_{56})	$0.40^{+0.96}_{-0.28}$	211 ± 9	Grove and Harrison (1996)
Muscovite *	$0.033^{+0.213}_{-0.029}$	183 ± 38	Hames and Bowring (1994)
Hornblende	$0.06^{+0.4}_{-0.01}$	276 ± 17	Harrison (1981)

Now been updated by Harrison et al., 2009

From Braun et al., Quantitative Thermochronology activity TIME

What approximations are involved in using T_C to link age to temperature?

Linking $^{40}Ar/^{39}Ar$ ages to temperature: assumptions



- No initial Ar in grain

Linking $^{40}Ar/^{39}Ar$ ages to temperature: assumptions

K K K K K K K K K K K K K K K
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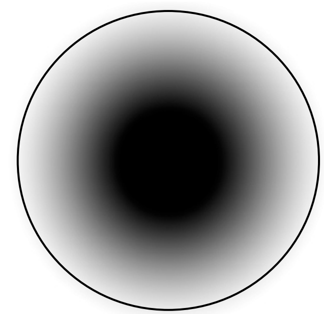
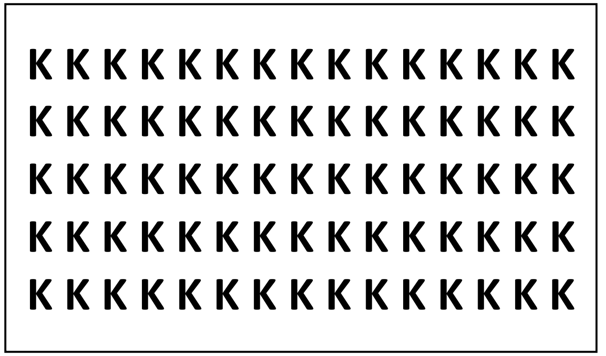
K K K K K K K K K K K K K K K
K K K K K K K K K K K K K K K
K K K K K K K K K K K K K K K

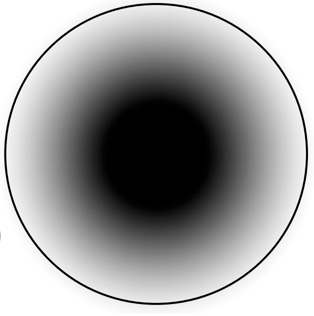
- No initial Ar in grain
- Thermally-activated volume diffusion

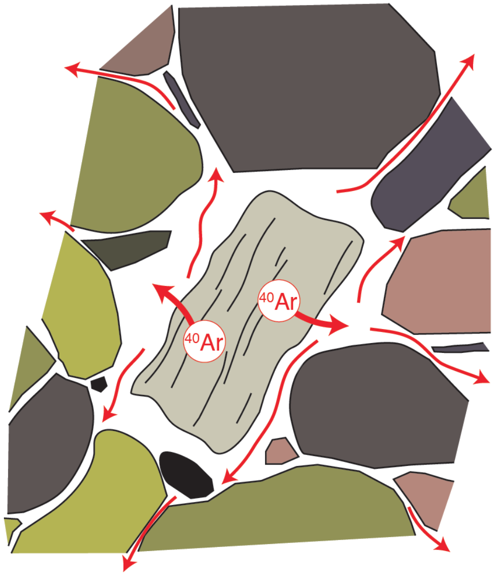


Linking $^{40}Ar/^{39}Ar$ ages to temperature: assumptions



- No initial Ar in grain
- Thermally-activated volume diffusion
- Infinite grain boundary reservoir (open system)





A. Smye

Linking ^{40}Ar/^{39}Ar ages to temperature: assumptions

K K K K K K K K K K K K K K K
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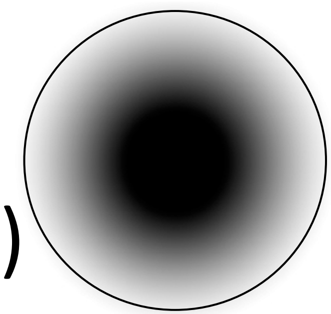
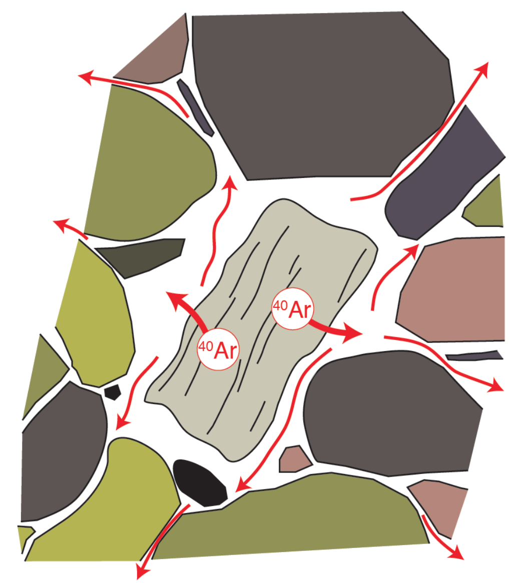
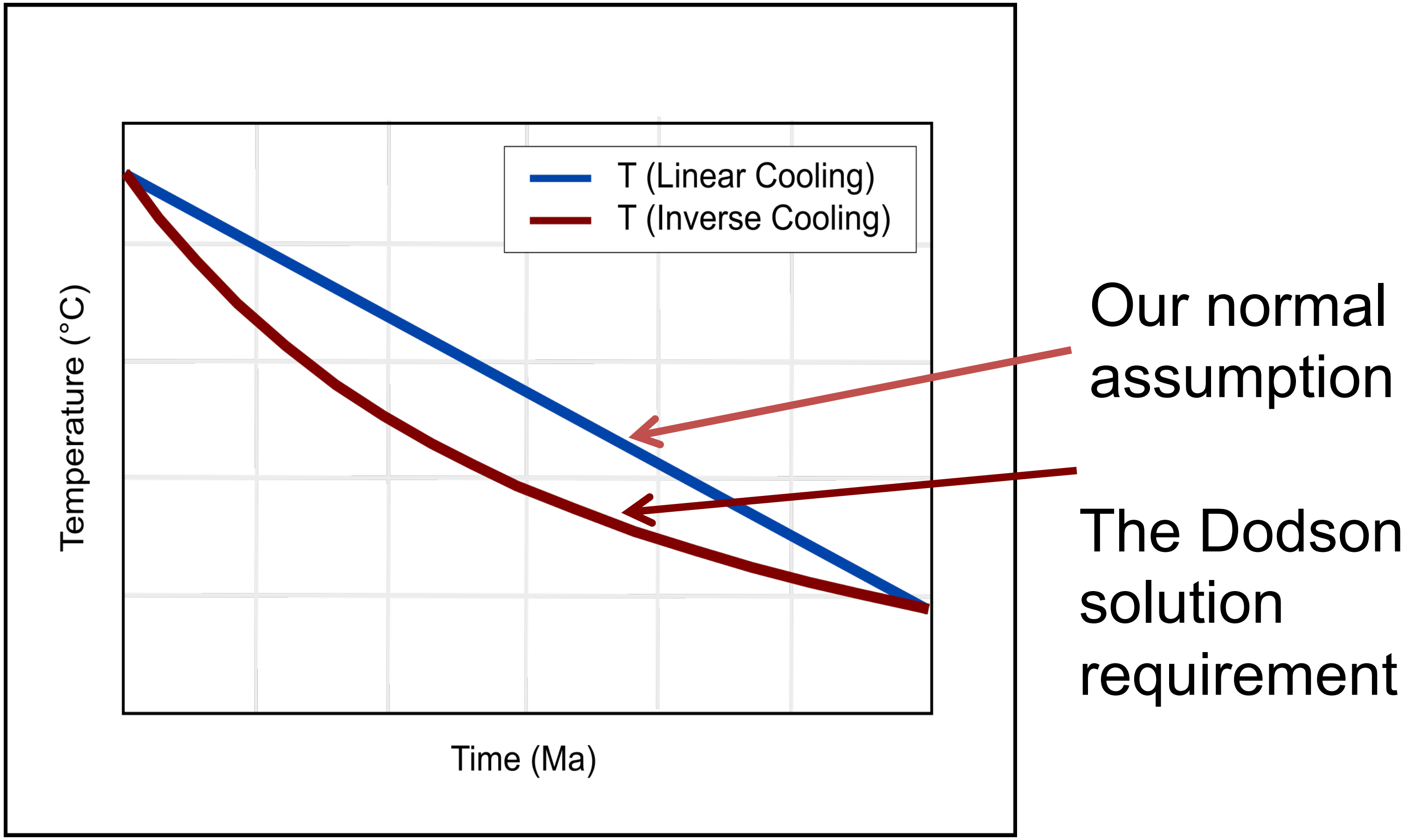
- No initial Ar in grain
- Thermally-activated volume diffusion
- Infinite grain boundary reservoir (open system)
- $T_{crystallisation}$ >> $T_{closure}$





A. Smye

Dodson: Specific 1/T cooling path



Conditions for the Dodson solution

- Initially, concentration = 0

$$\frac{\partial c}{\partial t} = D\nabla^2 c + S$$

- Where does the time dependence sit?

Conditions for the Dodson solution

- Initially, concentration = 0

$$\frac{\partial c}{\partial t} = D\nabla^2 c + S$$

- Where does the time dependence sit?
  - In S (source term)
  - In $D = D_0 \exp(-E_a / RT)$ (temperature linked to time)
  - Analytical solution possible if E_a/RT is linear with time
  - i.e a 1/time temperature dependence (messy!)

Linking $^{40}Ar/^{39}Ar$ ages to temperature: assumptions

K K K K K K K K K K K K K K K
K K K K K K K K K K K K K K K
K K K K K K K K K K K K K K K
K K K K K K K K K K K K K K K
K K K K K K K K K K K K K K K

- No initial Ar in grain
- Thermally-activated

Shouldn't use Dodson's formulation
unless these assumptions are satisfied

How can we tell? That's up next…

A. Smye

Learning Outcomes

- You will gain an understanding of:
  - Diffusion in minerals and rocks
  - Closure temperature
  - The assumptions behind the Dodson T_C formulation
- You will be able to:
  - Calculate closure temperatures